14.05 Intermediate Applied Macroeconomics
Problem Set 4

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Question 1 Diamond Overlapping Generations Model

Consider the Diamond overlapping generations model. L_t individuals are born in period t and live for two periods, working and saving in the first and living off capital in the second period. Assume population is growing at a constant rate, n, and technological progress occurs at exogenous rate g. Markets are competitive and labor and capital are paid their marginal products. There is no capital depreciation. Utility is logarithmic with individual discount rate $0 > \rho > -1$.

$$U = \log(c_t) + \frac{1}{1+\rho}\ \log(c_{t+1}).$$

The production function in per capita terms is

$$y_t = f(k_t) = k_t^{\alpha}.$$

(a) Determine the intertemporal budget constraint for each individual. Set up the consumer's utility maximization problem and derive the equilibrium condition for $\frac{c_{t+1}}{c_t}$ (Euler equation). Use that condition and the budget constraint to solve for first period consumption, c_t, and the savings rate (ie, the fraction of income saved), s_t.

(b) Using the saving rate derived in part (a) and the production function determine the relationship between k_{t+1} and k_t, and show it in a graph. Write down the expression that implicitly defines the equilibrium capital stock k^*. Is the equilibrium stable?

(c) Describe how each of the following affects the relationship between k_{t+1} and k_t. Show it in a graph and explain intuitively.

  (i) A rise in the population growth rate n.

  (ii) A decrease in the individual discount rate ρ.

  (iii) A downward shift of the production function. In particular, assume that $f(k_t)$ takes the form Bk_t^{α} and B falls.

(d) Now suppose that capital depreciates at rate $\delta > 0$, so that $r_t = f'(k_t) - \delta$. How, if at all, does this affect the savings rate derived in (a)? How does this result depend on the assumption of logarithmic utility?

Question 2 Social Security using Taxes

Consider a Diamond economy where g is zero, production is Cobb-Douglas, population grows at a rate n (from one generation to the next), and utility is logarithmic (see sections 2.9 and 2.10 in the textbook). Suppose the government taxes each young individual an amount T and uses the profits to pay benefits to old individuals; thus each old person receives $(1+n)T$.

(a) Write down the budget constraint faced by an individual born at time t.

(b) Use the budget constraint to determine the saving function of this individual. [*Hint: You should get a function that depends in the wages and the social security tax in the following way:* $s_t = Bw_t - Z_tT$ *where both* B *and* Z_t *are functions of the parameters of the model* (n, ρ, A) *and the interest rate at* $t+1$ (r_{t+1}) *.*]

(c) How, if at all, does the introduction of the pay as you go social security affect the relation between k_t and k_{t+1} as described by equation 2.59 in the textbook? [*Hint: Use the individual savings function obtained in part (b) and proceed as usual to determine the relationship between* k_t *and* k_{t+1}*.*]

(d) How, if at all, does the introduction of this social security system affect the balanced growth path value of k? [*Hint: The key is to determine the sign of* Z_t*.*]

(e) If the economy is initially on a balanced growth path (BGP) that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations? What happens if the initial BGP is dynamically inefficient? [*Hint: Remember that dynamic efficiency means that* $k^* = k(goldenrule)$*.*]

Question 3 Social Security and Personal Savings Accounts

Consider a Diamond economy where g is zero, production is Cobb-Douglas, population grows at a rate n (from one generation to the next), and utility is logarithmic (see sections 2.9 and 2.10 in the textbook). Suppose the government taxes each young individual an amount T and uses the proceeds to purchase capital. Individuals born at T therefore receive $(1+r_{t+1})T$ when they are old.

(a) Write down the budget constraint faced by an individual born at time t.

(b) Use the budget constraint to determine the saving function of this individual. [*Hint: You should again get a function that depends in the wages and the social security taxin the following way:* $s_t = Bw_t - Z_t T$ *where both* B *and* Z_t *are functions of the parameters of the model* (n, ρ, A) *and the interest rate at* $t+1$ (r_{t+1}).]

(c) How, if at all, does the introduction of the pay as you go social security affect the relation between k_t and k_{t+1} as described by equation 2.59 in the textbook? [*Hint: Use the individual savings function obtained in part (b) and proceed as usual to determine the relationship between* k_t *and* k_{t+1}.]

(d) How, if at all, does the introduction of this social security system affect the balanced growth path value of k?

(e) Compare the results obtained in questions 2.d and 3.d. Explain *intuitively* why the results are different.