Lecture 16

Incomplete Information
Static Case

14.12 Game Theory

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Road Map

1. Bayesian Nash Equilibrium
2. Examples
3. Cournot Duopoly
4. Quiz
5. Mixed strategies, revisited

Bayesian Game (Normal Form)

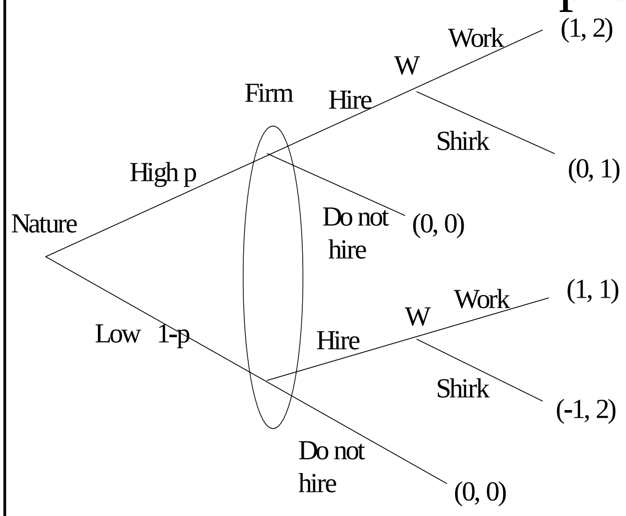
A Bayesian game is a list

$G = \{A_1,\ldots,A_n;T_1,\ldots,T_n;p_1,\ldots,p_n;u_1,\ldots,u_n\}$

where

- A_i is the action space of i (a_i in A_i)
- T_i is the type space of i (t_i)
- $p_i(t_{-i}|t_i)$ is i's belief about the other players
- $u_i(a_1,\ldots,a_n;t_1,\ldots,t_n)$ is i's payoff.

An Example



$T_{Firm}=\{t_f\}$;
$T_W = \{High,Low\}$
$A_{Firm} = \{Hire, Don't\}$
$A_W = \{Work,Shirk\}$
$p_F(High) = p$
$p_F(Low) = 1-p$

Bayesian Nash equilibrium

A Bayesian Nash equilibrium is a Nash equilibrium of a Bayesian game.

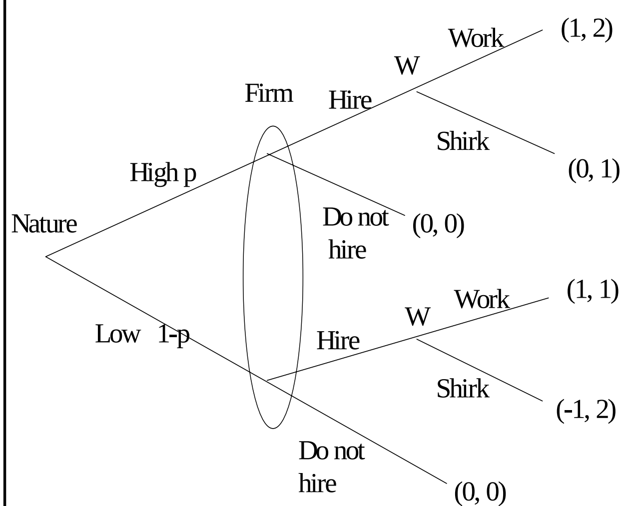
Given any Bayesian game G = $\{A_1,\ldots,A_n;T_1,\ldots,T_n;p_1,\ldots,p_n;u_1,\ldots,u_n\}$

a **strategy** of a player i in a is any function $s_i:T_i \rightarrow A_i$;

A strategy profile s* = $(s_1^*,\ldots, s_1^*)$ is a **Bayesian Nash equilibrium** iff $s_i^*(t_i)$ is a best response to s_{-i}^* for each t_i, i.e., $s_i^*(t_i)$ solves

$$\max_{a_i \in A_i} \sum_{t_{-i} \in T_{-i}} u_i\left(s_1^*(t_1),\ldots,s_{i-1}^*(t_{i-1}),a_i,s_{i+1}^*(t_{i+1}),\ldots,s_n^*(t_n);t\right)p_i(t_{-i} \mid t_i)$$

An Example



$T_{Firm}=\{t_f\}$;
T_W = {High,Low}
A_{Firm} = {Hire, Don't}
A_W = {Work,Shirk}
p_F(High) = p >1/2
p_F(Low) = 1-p

s_F^* = Hire
s_W^* (High) = Work
s_W^* (Low) = Shirk

Another equilibrium?

Another example

	L	R
X	θ,γ	1,2
Y	$-1,\gamma$	θ,0

- $\theta \in \{0,2\}$, known by Player 1
- $\gamma \in \{1,3\}$, known by Player 1
- All values are equally likely
- $T_1 = \{0,2\}$; $T_2 = \{1,3\}$
- Bayesian Nash Equilibrium:
- $s_1(0) = s_1(2) = X$
- $s_2(1) = R$; $s_2(3) = L$

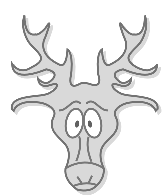
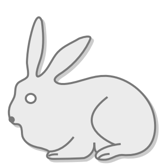
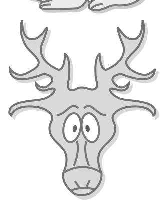
Cournot Duopoly with Incomplete Info

- $N = \{1,2\}$ firms;
- Price: $P = 1- (q_1+q_2)$
- Marginal cost of Firm 1 is c=0.
- Marginal cost of Firm 2 is
  - c_H with probability θ,
  - c_L with probability $1-\theta$.
- Firm 2 knows its own marginal cost.
- Strategies: q_1; $(q_2(c_H),q_2(c_L))$

Cournot Duopoly - BNE

- $q_1 = [\theta(1-q_2(c_H)) + (1-\theta)(1-q_2(c_L))]/2;$
- $q_2(c_H) = (1-q_1-c_H)/2$
- $q_2(c_L) = (1-q_1-c_L)/2$
- $q_1^* = (1 + \theta c_H + (1-\theta)c_L)/3$
- $q_2^*(c_H) = (1-2c_H)/3 + (1-\theta)(c_H-c_L)/6$
- $q_2^*(c_L) = (1-2c_L)/3 - \theta(c_H-c_L)/6$

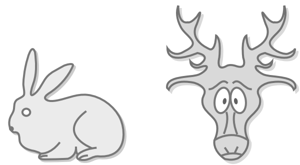
Stag Hunt, Mixed Strategy

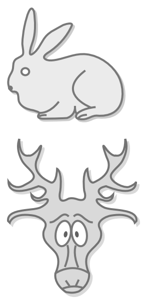





(rabbit)	(2,2)	(2,0)
(stag)	(0,2)	(4,4)

Figures by MIT OCW.

Mixed Strategies





2+t,2+v	2+t,0
0,2+v	4,4

$U_1(R|t) = 2+t$
$U_1(S|t) = 4(1-q)$;
$U_1(R|t) > U_1(S|t) \Leftrightarrow t > 0$

- t and v are iid with uniform on $[-\varepsilon,\varepsilon]$.
- t and v are privately known by 1 and 2, respectively.
- Pure strategy:
  - $s_1(t)$ = Rabbit iff $t > 0$;
  - $s_2(v)$ = Rabbit iff $t>0$.
- $p = \text{Prob}(s_1(t)=\text{Rabbit}|v) = \text{Prob}(t > 0) = 1/2$.
- $q = \text{Prob}(s_2(v)=\text{Rabbit}|t) = 1/2$

Figures by MIT OCW.