Recitation Three Problems

Problem One

There is an interesting scene in *A Beautiful Mind* in which John Nash's character discovers his equilibrium concept. The setup is that there are four men and five women at a bar, and each man must simultaneously walk over to a woman. One of the women, who is blonde, is considered to be more desirable than the other four, who are all brunettes. Although the men all prefer the blonde to the brunettes, if they all go for the blonde they will "block each other" (according to the logic of the movie) and end up unsuccessful; moreover, after going for the blonde, they cannot then go to a brunette because the brunette would be offended at being someone's second choice and would turn down the man. Nash's character then realizes that what ought to happen is for each of the four men to choose a brunette. Let's try to put this situation in a game theoretic model by using a to denote the payoff a man would receive if another man goes for the same woman he does (here there are assumptions that it is possible to "block each other" on brunettes and that being "blocked" on a brunette gives the same payoff as being "blocked" on a blonde, although this is not important for the problem). We'll use b to refer to a man's payoff if he is the only man to go for a woman and that woman is a brunette. We'll use c to refer to a man's payoff if he is the only man to go for a woman and that woman is blonde. The natural restrictions on payoffs are $a<b<c$.

Is there a pure strategy Nash equilibrium where each man ends up with a brunette? What are the pure strategy Nash equilbria? How do these answers change if the blonde turns down everyone (meaning that the payoff to going to the blonde is a, regardless of how many men choose her)?

Problem Two

Find the pure strategy Nash equilibria and the rationalizable strategies in the following game:

1 / 2	L	C	R
U	5,5	0,0	5,2
M	0,0	4,4	5,2
D	2,5	2,5	3,3

Problem Three

Two MIT students who have taken 14.12 live together in a messy apartment and have exactly 24 hours before company is coming over. They're not going to be able to make the apartment as clean as they want in the next 24 hours even if both roommates clean nonstop, but each student would like the apartment to be as clean as possible. The students are equally productive at cleaning the apartment, and there is a disutility of effort. In particular, assume that the payoff for player 1 is $3 * \sqrt[3]{h_1 h_2} - h_1$. The payoff for player 2 is symmetric. To decide how much time each student will spend cleaning, they decide they are going to each simultaneously write down some amount of time on a piece of paper saying how much time he will spend cleaning, and suppose that each student is bound to the amount of time he writes. What are the Nash equilibria, and what are the rationalizable strategies?