

## Exam 2

14.30, Fall 2000

This exam is closed-notes and closed-book, although you may bring in one page of notes. You may use a calculator. Please consult the third page of this exam for notation, PMFs/PDFs, means, and variances of special distributions.

READ THROUGH THE EXAM FIRST, in order to ask clarifying questions and to allocate your time appropriately. In order to receive partial credit in the case of computational errors, please show all work. You have approximately 85 minutes in which to complete the exam. Good luck!

1. (25 pts.) True or False with short explanation.

a)  $Y = \log(Z)$  where  $Z \sim \mathcal{N}(0, 1)$  has a lognormal distribution.

b) A  $\mathcal{B}(n, p)$  random variable is the sum of  $n$  *i.i.d.* Bernoulli random variables.

c) A  $\mathcal{NB}(r, p)$  random variable is the difference between two independent  $\mathcal{B}(r, p)$  random variables.

d) A Poisson random variable can be interpreted as the waiting times between relatively rare events.

e) If  $X \sim \mathcal{B}(n, p)$ ,  $A = [a_1, a_2]$ , and  $\Phi(\cdot)$  is the standard normal CDF, then

$$\lim_{n \rightarrow \infty} \mathbb{P}\left(\frac{X - np}{\sqrt{np(1-p)}} \in A\right) = \Phi(a_2) - \Phi(a_1).$$

**2. (24 pts.)**

a) What is the distribution of  $Y$  where  $X \sim \mathcal{N}(0, 1)$  and

$$Y = \begin{cases} -1 & \text{if } X \leq -1/2 \\ 0 & \text{if } -1/2 < X \leq 1/2 \\ 1 & \text{if } X > 1/2 \end{cases}$$

b) What is the joint distribution of  $Z_i$ ,  $i = 1, \dots, n$  where  $U_i$  *i.i.d.*  $\mathcal{U}(0, 1)$  and  $Z_i = (n - i + 1)U_{n-i+1}$ . (You may use the fact that  $g_i(X_i)$  are independent if the  $X_i$  are independent, if you would like, but it is not necessary.)

**3. (26 pts.)** In our weekly squash game, Professor Duflo and I play for a random length of time in hours  $T$ , where  $T \sim \mathcal{E}(5/4)$ . The number of really good shots I make,  $S$ , has a distribution that depends on the amount of time we play, in particular  $S \sim \mathcal{P}(10T)$ . Finally, the number of complete games we are able to fit in is  $N = \text{int}(4T)$ , where  $\text{int}(\cdot)$  is the greatest integer function.

a) What is the expected number of really good shots I make conditional on  $T$ ? What is the conditional variance?

b) What is the unconditional expectation and variance of  $S$ ?

c) What is the unconditional probability that I hit no really good shots?

d) What is the expected number of complete games we play?

Note: Parts c) and d) are fairly difficult. Please do not spend much time on them until (if) you've finished the rest of the exam.

**4. (25 pts.)** To ensure combat readiness, the U.S. Navy has its Tomcat pilots on aircraft carriers fly  $n$  practice flights each week. Suppose a particular pilot's probability of landing

successfully on the carrier upon her first approach is 80%, and that each of her flights is independent. Below are the numbers of successful landings she has had upon first approach for the past ten weeks.

$$\{4, 3, 3, 5, 4, 1, 4, 3, 5, 5\}$$

a) Propose two different (reasonable) estimators for  $n$ . What are the estimates that each produce, given the sample above?

b) If the true number of flights each week is 6, what are the probabilities (in general, not based on the sample above) that each of your estimators is within 0.5 of the true value given 30 observations? (You may use appropriate approximation techniques.)